Assignment 5

Algorithm Design and Analysis

December 6, 2015

Notice:

1. **Due** 9:00 a.m., Dec. 18 for hard copy and 11:55 p.m., Dec. 18 for digital version, 2015 for graduate students at UCAS;
2. Please submit your answers in hard copy **AND** submit a digital version to UCAS website https://www2.ucas.ac.cn/ .
3. Please choose **at least 4** problems from Problem 1-7 and **at least 2** problems from Problem 8-10.
4. When you're asked to give an algorithm, you should do at least the following things:
   - Describe the basic idea of your algorithm in natural language **AND** pseudo-code;
   - Prove the correctness of your algorithm.
   - Analyse the complexity of your algorithm.

1 Problem Reduction

Support the you are a matchmaker and there are some boys and girls. Since the boys are alway more than girls, you can assume that if a girl express her love to a boy , the boy will always accept her. Now you know every girl's thought(a girl may like more than one boy) and you want to make as much pairs as you can. show that you can do this using maximum flow algorithm.

2 Problem Reduction

There is a matrix filled with 0 and 1, you know the sum of every row and column. you are asked to give such a matrix which satisfy the conditions.

3 Unique Cut

Let $G = (V, E)$ be a directed graph, with source $s \in V$, sink $t \in V$, and nonnegative edge capacities c_e. Give a polynomial-time algorithm to decide whether G has a unique minimum st cut.

4 Problem Reduction

There is a matrix with numbers which means the cost when you walk through this point. you are asked to walk through the matrix from the top left point to the right bottom point and then return to the top left point with the minimal cost. note that when you walk from the top to the bottom you can just walk to the right or bottom point and when you return, you can just walk to the top or left point. And each point CAN NO be walk through more than once.

5 Dogs and kennels

On a grid map there are n little dogs and n kennels. In each unit time, every little dog can move one unit step, either horizontally, or vertically, to an adjacent point. For each little dog, you need to pay a 1 travel fee for every step it moves, until it enters a kennel. The task is complicated with the restriction that each kennel can accommodate only one little dog.

Give a polynomial-time algorithm to compute the minimum amount of money you need to pay in order to send these n little dogs into those n different kennels.

6 Maximum Cohesiveness

Given an undirected graph, each edge is assigned one weight, find a subset S of nodes to maximize $e(S)/|S|$, where $e(S)$ denotes the sum of edge weights in S and $|S|$ is the number of nodes in S. Give a polynomial-time algorithm that takes a rational number α and de- termines whether there exists a set S with cohesiveness at least α.

7 Maximum flow

Another way to formulate the maximum-flow problem as a linear program is via flow decomposition. Suppose we consider all (exponentially many) s-t paths P in the network G, and let fP be the amount of flow on path P. Then maximum flow says to find

$$\begin{array}{rrcl} \max & z & = & \sum_{e \in p} f_p \\ s.t. & f_p & \leq & u_e, \quad for \quad all \quad edge \quad e \\ & f_p & \geq & 0 \end{array}$$

(The first constraint says that the total flow on all paths through e must be less than ue.) Take the dual of this linear program and give an English explanation of the objective and constraints.

8 Ford-Fulkerson algorithm

Implement Ford-Fulkerson algorithm to find the maximum flow of the following network, and list your intermediate steps. Use you implementation to solve problem 1 and show your answers.
INPUT: Number of girls and boys. the boys that each girl likes. see more detial in the file problem1.data.
OUTPUT: Number of pairs you can make.

9 Push-relabel

Implement push-relabel algorithm to find the maximum flow of a network, and list your intermediate steps. Use your implementation to solve problem 2 and write a check problem to see if your answer is right.
INPUT: Numbers of rows and columns. And the sum of them. See more detial in the file problem2.data.
OUTPUT: The matrix you get. Any one satisfy the conditions will be accept.

10 Cycle canceling

Implement Cycle canceling algorithm to find the minimum cost flow of a network, and list your intermediate steps.
INPUT: A directed graph $G =< V, E >$. Each edge e has a capacity c_e and a cost w_e. Two special points: source s and sink t. Please make the input in DIMACS format. For DIMACS format, see DIMACS maximum flow problems.html in this folder.
OUTPUT: For each edge e, to assign a flow f_e such that $\sum_{e \in E} f_e w_e$ is minimized.