

# Assignment 3

## Algorithm Design and Analysis

October 24, 2015

Notice:

1.     • **Due** 9:00 a.m., Nov. 6, 2015 for graduate students in UCAS;
2. Please submit your answers in hard copy AND submit a digital version to UCAS website <https://www2ucas.ac.cn/>.
3. Please choose at least two problems from Problem 1-4, and choose at least one problem from Problem 5-6.
4. When you're asked to give an algorithm, you should do at least the following things:
  - Describe the basic idea of your algorithm in natural language **AND** pseudo-code;
  - Prove the correctness of your algorithm.
  - Analyse the complexity of your algorithm.

## 1 Greedy Algorithm

Given a list of  $n$  natural numbers  $d_1, d_2, \dots, d_n$ , show how to decide in polynomial time whether there exists an undirected graph  $G = (V, E)$  whose node degrees are precisely the numbers  $d_1, d_2, \dots, d_n$ .  $G$  should not contain multiple edges between the same pair of nodes, or “loop” edges with both endpoints equal to the same node.

## 2 Greedy Algorithm

There are  $n$  distinct jobs, labeled  $J_1, J_2, \dots, J_n$ , which can be performed completely independently of one another. Each job consists of two stages: first it needs to be *preprocessed* on the supercomputer, and then it needs to be *finished* on one of the PCs. Let's say that job  $J_i$  needs  $p_i$  seconds of time on the supercomputer, followed by  $f_i$  seconds of time on a PC. Since there are at least  $n$  PCs available on the premises, the finishing of the jobs can be performed on PCs at the same time. However, the supercomputer can only work on a single job a time without any interruption. For every job, as soon as the preprocessing is done on the supercomputer, it can be handed off to a PC for finishing.

Let's say that a *schedule* is an ordering of the jobs for the supercomputer, and the *completion time* of the schedule is the earliest time at which all jobs have finished processing on the PCs. Give a polynomial-time algorithm that finds a schedule with as small a completion time as possible.

### 3 Greedy Algorithm

In a party, a host ask  $n$  boys and  $n$  girls to play a game, in which a boy and a girl have to walk towards the finish line while keeping a balloon staying between their heads. This game needs cooperation, but height difference between the two players is also important. Suppose the height of boys are  $b_1, b_2, \dots, b_n$  and the height of girls are  $g_1, g_2, \dots, g_n$ . The problem is to match  $n$  girls and  $n$  boys such that the *average difference* between the corresponding players is minimized. That is, you want to minimize  $\frac{1}{n} \sum_{i=1}^n |b_i - g_i|$ . Give a polynomial-time algorithm to solve this problem.

### 4 Greedy Algorithm

Suppose you are given two sets  $A$  and  $B$ , each containing  $n$  positive integers. You can choose to reorder each set however you like. After reordering, let  $a_i$  be the  $i$ th element of set  $A$ , and let  $b_i$  be the  $i$ th element of set  $B$ . You then receive a payoff of  $\prod_{i=1}^n a_i^{b_i}$ . Give an polynomial-time algorithm that will maximize your payoff.

### 5 Programming

Write a program in your favorite language to compress a file using Huffman code and then decompress it. Code information may be contained in the compressed file if you can. Use your program to compress the two files (*graph.txt* and *Aesop\_Fables.txt*) and compare the results (Huffman code and compression ratio).

### 6 Programming

1. Implement Dijkstra's algorithm (using linked list, binary heap, binomial heap, and Fibonacci heap) to calculate the shortest path from node  $s$  to node  $t$  of the given graph (*graph.txt*), where  $s$  and  $t$  are randomly chosen. The comparison of different priority queue is expected.

*Note: you can implement the heaps by yourself or using Boost C++/STL, etc.*

2. Figure out how many shortest paths is every node lying on in your program, except starting node  $s$  and finishing node  $t$ . For example, if there are in total three shortest paths  $0 \rightarrow 1 \rightarrow 2 \rightarrow 10$ ,  $0 \rightarrow 1 \rightarrow 3 \rightarrow 4 \rightarrow 10$  and  $0 \rightarrow 1 \rightarrow 2 \rightarrow 6 \rightarrow 7 \rightarrow 10$ , then 1 lies on 3 shortest paths, 2 lies on 2 shortest paths, and 3 lies on 1 shortest path, etc.