Assignment 2

Chinese Academy of Sciences

October 10, 2015

Notice:

1. Deadline of handing in both your hard copy and digital version: 9 am. on Oct. 23, 2015.
2. You must submit both on time, or we can't give the score for this assignment.
3. You can choose 3 problems from Problem 1-5, and Problem 6 is required.
4. You can present your answer in English or in Chinese, but pdf complied from .tex file is strongly recommended.
5. In Problem 6, you should implement your algorithm in C/C++/Java with good comments.
6. In Problem 1-5, you should do at least the following things: 1) Describe the optimal subproblems and DP equation; describe your algorithm in daily language or pseudo-code; 2) Prove that your algorithm can give the right answer; 3) Analyse the complexity of your algorithm.
7. This assignment will contribute 3 marks to your final score. If you don't hand in your answer sheet in due time or your answer sheet is a duplicate of others, both of you will get 0; if you hand in your answer sheet in due time but write nearly nothing, you will get 1; if your assigment doesn't contain any source code, you will get at most 2.

1 Money robbing

A robber planning to rob houses along a street. Each house has a certain amount of money stashed, the only constraint stopping you from robbing each of them is that adjacent houses have security system connected and it will automatically contact the police if two adjacent houses were broken into on the same night.

1. Given a list of non-negative integers representing the amount of money of each house, determine the maximum amount of money you can rob tonight without alerting the police.

2. What if all houses are arranged in a circle?

2 Minimum path sum

Find the minimum path sum of a triangle from top to bottom. Each step you may move to adjacent numbers on the row below.

For example, given the following triangle:

$$
\begin{array}{c}
2 \\
3\ 4 \\
6\ 5\ 7 \\
4\ 1\ 8\ 3
\end{array}
$$

The minimum path sum from top to bottom is 11 (**i.e.**, $2+3+5+1=11$).

3 Partition

Given a string s, partition s such that every substring of the partition is a palindrome. Return the minimum cuts needed for a palindrome partitioning of s.

For example, given $s =$ "aab", return 1 since the palindrome partitioning ["aa", "b"] could be produced using 1 cut.

4 Subsequence Counting

Given a string S and a string T, count the number of distinct subsequences of T in S.

A subsequence of a string is a new string which is formed from the original string by deleting some (can be none) of the characters without disturbing the relative positions of the remaining characters. (ie, "ACE" is a subsequence of "ABCDE" while "AEC" is not).

Here is an example: S = "rabbbit", T = "rabbit". Return 3.

5 Decoding

A message containing letters from A-Z is being encoded to numbers using the following mapping:

A: 1
B: 2
...
Z: 26

Given an encoded message containing digits, determine the total number of ways to decode it.

For example, given encoded message "12", it could be decoded as "AB" (1 2) or "L" (12). The number of ways decoding "12" is 2.

6 Maximum profit of transactions

Say you have an array for which the i-th element is the price of a given stock on day i.

Design an algorithm and implement it to find the maximum profit. You may complete at most two transactions.

Note: You may not engage in multiple transactions at the same time (ie, you must sell the stock before you buy again).